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One Approach to the Assessment of Bilingual Education

Abstract.
It is formulated and tested a new approach to the estimation of bilingual education for students, which is based on the fuzzy inferences method. For alternative estimation of bilingual classes it is selected block of assessment criterions by which on the basis of appropriate marks of observations it is made an assessment of an audience as a whole, the individual student and the teacher.

Keywords: bilingual education, assessment criterion, fuzzy set, fuzzy conclusion

1. Introduction
Bilingual education integrating a substantial part of learning and natural language is, in a sense, a very “sensitive” and “delicate” process. During its implementation and follow-up realization it is very important do not impair the current knowledge and the advancing progress of the student, his intellectual potential and innate abilities. Therefore, from a methodological point of view there are two obvious and fundamental problems: how to provide the necessary academic level of student's knowledge when he (she) studies on a second (non-native) language and how to assess adequately the level of acquirement in a particular subject and its progress in mastering a second language?

Under exist evaluation methods it is almost impossible separately certify the student on language and substantive components of learning. Herewith it is
possible a misunderstanding, when the lack of lexical dictionary of student is interpreted for the benefit of his failure to fully understand of the essence of the subject. Obviously, when bilingual education would be wise to evaluate the student by two specialists: for informal theory by teacher of a subject, and for language by the second language teacher. However, this approach is still artificial, illogical and ultimately does not attract the interest of the student.

Moreover, the parallel evaluation of a bilingual education is unacceptable and other very important (from methodological point of view) position. In subjects where language training is not important (for example, in mathematics) student easily solving the problem, because of the lack of knowledge of the language can not correctly interpret the problem formulation, especially if it is not clearly formulated or presence of “new words”. For example, improperly imprinted comma can radically change the essence of the sentence.

Testing, which is currently used to evaluate the bilingual education in pilot educational institutions, uses simplified statements in tests involving a unambiguous choice of answer on principle "YES – NO". Unfortunately this method provides only an approximate proposition of the true of knowledge level of students, especially as in the process of testing it is an element of chance, and thus do not provide full objectivity. Therefore, only the system of alternative estimation is able objectively to assess the level of acquired language skills in the context of learning the basic didactic material for a course.

As one of the alternative methods of assessment of students in bilingual education it is proposed using a fuzzy inferences mechanism, which is by far one of the best methods of multi-criteria evaluation of alternatives under uncertainty. Due to this approach along with the usual numbers it is possible to involve nonmetrizable (or semi-structured) data in computational process, that
is very important for estimation the quality of bilingual education (Zadeh, 1976).

2. Problem formulation

Suppose that at a certain discipline it is bilingual class characterized by linguistic heterogeneity of the audience. For an alternative assessment of this class let’s choose a block of assessment criterions consisting following four parts (Aliev & Kazhe, 2005):

- assessment of the audience as a whole;
- assessment of the student during the integrated study;
- assessment of the teacher;
- options for self-conducted studies by teacher.

Then on the basis of these criteria it is necessary to create a method of alternate assessment based on the application of fuzzy logic inference mechanism under inaccuracies and vagueness of available information, and, thus, to obtain the aggregated assessment of bilingual class.

Table 1. Assessment of the audience as a whole

<table>
<thead>
<tr>
<th>I part</th>
</tr>
</thead>
<tbody>
<tr>
<td>Teacher’s name</td>
</tr>
<tr>
<td>Date of observation</td>
</tr>
<tr>
<td>The number of students in the classroom</td>
</tr>
<tr>
<td>Time of observation</td>
</tr>
<tr>
<td>Name of the observer</td>
</tr>
<tr>
<td>Aggregated assessment</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>II part (activity at work in subgroups estimated by the 10-point scale)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Symbolic notation of group</td>
</tr>
<tr>
<td>-----------------------------</td>
</tr>
<tr>
<td>$a_1$</td>
</tr>
<tr>
<td>$a_2$</td>
</tr>
<tr>
<td>...</td>
</tr>
<tr>
<td>$a_n$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>III part (working besides subgroups according to the special task)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
</tr>
<tr>
<td>Criterion</td>
</tr>
<tr>
<td>---------------------------</td>
</tr>
<tr>
<td>The number of students</td>
</tr>
</tbody>
</table>

Table 2. Score of student working in the subgroups

### Verbal action

<table>
<thead>
<tr>
<th>Symbolic notation</th>
<th>Expresses sentences (offers to cooperate) in both languages</th>
<th>Requests that work together – in both languages or only native language</th>
<th>Says as a “facilitating” actions, explain to others without their request – in both languages or only native language</th>
<th>Speaks during a joint operation (talking about cooperation) – in both languages or only native language</th>
<th>Speaks during a joint operation (speaking extraneous subject) in native language</th>
</tr>
</thead>
<tbody>
<tr>
<td>u₁</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>u₂</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>u₃</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>u₄</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### Actions or behavior

<table>
<thead>
<tr>
<th>Symbolic notation</th>
<th>Works alone</th>
<th>Works with others</th>
<th>Acts as facilitating the work (performs most of it)</th>
<th>Learns (listens)</th>
<th>Waits for help from the outside</th>
<th>Oscillates between the desire to do and search for help</th>
<th>Reads something, perhaps is not related with a theme of classes (usually in the native language)</th>
<th>Idles, don’t want to work</th>
</tr>
</thead>
<tbody>
<tr>
<td>u₁</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>u₂</td>
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<tr>
<td>u₃</td>
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<tr>
<td>u₄</td>
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<td></td>
<td></td>
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<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 3. Evaluation of the teacher

<table>
<thead>
<tr>
<th>#</th>
<th>Activities</th>
<th>Cases</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Facilitates the task of the student</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>Makes a comment about the discipline (the student or the audience in whole)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>Informs, instructs, defines</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>Asks questions on the subject</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>Promotes higher-order thinking</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>Identifies and insonifies the interdisciplinary connections</td>
<td></td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>Provides additional information (materials) to a group or an student</td>
<td></td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>Justifies the need for joint action</td>
<td></td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>Improves the degree of competence</td>
<td></td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>Indicates the need of variety of roles in the group</td>
<td></td>
<td></td>
</tr>
<tr>
<td>11</td>
<td>Explains why one person is unable to perform the entire task the proposed group</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
3. Multi-criteria Choice of Alternatives with Using the Rule of Fuzzy Conclusion

Let $U$ is a set of alternatives (universal set), and $\tilde{A}$ is its fuzzy subset the accessory to which elements from $U$ is defined by corresponding values from $[0, 1]$ of membership function. Assume that fuzzy sets $\tilde{A}_j$ describe possible values (terms) of a linguistic variable $x$. Then, the set of decisions (alternatives) can be characterized by set of criteria – values of linguistic variables $x_1$, $x_2$, ..., $x_p$. For example, the value of linguistic variable $x_1$=“control quality” by the term “LOW”.

Set of the linguistic variables (criteria) accepting similar values can characterize representations about sufficiency of considered alternatives. Then, believing that $S$=SUFFICIENCY also as a linguistic variable the typical fuzzy implicative rule can look like

“If $x_1$=LOW and $x_2$=GOOD, then $S$=HIGH”.

Generally it is possible to present implicative reasoning in the following type [4]:

$$ e_i: \text{if } x_1=\tilde{A}_{1i} \text{ and } x_2=\tilde{A}_{2i} \text{ and } \ldots \text{ and } x_p=\tilde{A}_{pi}, \text{ then } S=\tilde{B}_i. \quad (1) $$

In particular, assuming that $x$ is a linguistic variable named as “control quality”, and $S$ is a linguistic variable named as «sufficiency of services», then in the notation of fuzzy implicative rules equality (Eq. 1.2) can look, for example, as:

\[
\begin{align*}
\text{If } x = \text{LOW}, \text{ then } S &= \text{SATISFACTORY}; \\
\text{If } x = \text{MORE THAN LOW}, \text{ then } S &= \text{LESS THAN SATISFACTORY}; \\
\text{If } x = \text{HIGH}, \text{ then } S &= \text{UNSATISFACTORY}.
\end{align*}
\]

Let's indicate the intersection of sets $x_1=\tilde{A}_{1i} \cap x_2=\tilde{A}_{2i} \cap \ldots \cap x_p=\tilde{A}_{pi}$ as $x=\tilde{A}_i$. In a discrete case operation of intersection of fuzzy sets is defined by a finding of a minimum of corresponding values of their membership functions [6, 8], i.e.

$$ \mu_{\tilde{A}_i}(v) = \min_{v \in V} (\mu_{\tilde{A}_{1i}}(u_1), \mu_{\tilde{A}_{2i}}(u_2), \ldots, \mu_{\tilde{A}_{pi}}(u_p)), \quad (2) $$

where $V=U_1 \times U_2 \times \ldots \times U_p$; $v=(u_1,u_2,\ldots,u_p)$;

$\mu_{\tilde{A}_i}(u_j)$ is the degree of accessory of element $u_j$ to fuzzy set $\tilde{A}_{ji}$. Then it is possible to
present statements (Eq. 1.2) in more compact type:

\[ e_i: \text{if } x = \bar{A}_i, \text{then } S = \bar{N}_i. \]

(3)

For the purpose of generalization of the given statements we will denote base sets \( U \) and \( V \) in the form of set \( W \). Then, \( \bar{A}_i \) will be a fuzzy subset of base set \( W \), and \( \bar{N}_i \) will be a fuzzy subset of an unit segment \( I = [0;1] \).

For realization of fuzzy logic rules it is used various fuzzy implication operations (see Table 1.1) [2, 9].

**Table 4: Fuzzy implication operations**

<table>
<thead>
<tr>
<th>No</th>
<th>Fuzzy implication name</th>
<th>Fuzzy implication operations</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>L. Zadeh</td>
<td>( I_m(x, y) = \max(1 - x, \min(x, y)) )</td>
</tr>
<tr>
<td>2</td>
<td>Lukasiewicz</td>
<td>( I_a(x, y) = \min(1, 1 - x + y) )</td>
</tr>
<tr>
<td>3</td>
<td>Minimum (Mamdani)</td>
<td>( I_c(x, y) = \min(x, y) )</td>
</tr>
<tr>
<td>4</td>
<td>Standard Star (Godel)</td>
<td>( I_g(x, y) = \begin{cases} 1, x \leq y \ y, x &gt; y \end{cases} )</td>
</tr>
<tr>
<td>5</td>
<td>Kleene – Dienes</td>
<td>( I_k(x, y) = \max(1 - x, y) )</td>
</tr>
<tr>
<td>6</td>
<td>Gaines</td>
<td>( I_\Delta(x, y) = \begin{cases} 1, x \leq y \ y, x &gt; y \end{cases} )</td>
</tr>
</tbody>
</table>

In the accepted designations let us choose Lukasevich’s implication:

\[
\mu_{\bar{H}}(w, i) = \min_{w \in W}(1, 1 - \mu_{\bar{A}}(w) + \mu_{\bar{N}}(i)),
\]

(4)

where \( \bar{H} \) is a subset on \( W \times I; w \in W \) and \( i \in I \).

Similarly reasonings (rules) \( e_1, e_2, \ldots, e_q \) are transposed in corresponding fuzzy sets \( \bar{H}_1, \bar{H}_2, \ldots, \bar{H}_q \). Thus, denoting their product as \( \bar{D} = \bar{H}_1 \cap \bar{H}_2 \cap \ldots \cap \bar{H}_q \) for each pair \((w, i) \in W \times I\) we have:

\[
\bar{G} = \bar{A} \circ \bar{D},
\]

(6)
where $\tilde{G}$ is a fuzzy subset of the unit segment $I$. Then as a result:

$$\mu_\tilde{G}(i) = \max_{w \in W} \{ \min \mu_A(w), \mu_B(w, i) \}.$$  

(7)

Comparison of alternatives is carried out on the basis of their point estimates. For this purpose at first for fuzzy subset $\hat{C} \subseteq I$ these are defined $\alpha$-level sets ($\alpha \in [0; 1]$) in the form of $C_\alpha = \{ i | \mu_C(i) \geq \alpha, i \in I \}$. Then, for each of them average values of corresponding elements $M(C_\alpha)$ are defined.

Generally for the set composed of $n$ elements we have:

$$M(C_\alpha) = \frac{1}{n} \sum_{j=1}^{n} i_j, \quad i_j \in C_\alpha.$$  

(8)

In particular, for $C_\alpha = \{ a \leq i \leq b \}$ $M(C_\alpha) = (a + b)/2$. In case of $0 \leq a_1 \leq b_1 \leq a_2 \leq b_2 \leq \ldots \leq a_n \leq b_n \leq 1$ and

$$C_\alpha = \bigcup_{j=1}^{n} \{ a_j \leq i \leq b_j \}$$

$$M(C_\alpha) = \frac{\sum_{j=1}^{n} a_j + b_j}{2} \frac{(b_j - a_j)}{\sum_{j=1}^{n} (b_j - a_j)}.$$  

(9)

As a result the point estimate of fuzzy set (alternative) $\hat{C}$ can be obtained from equality:

$$F(\hat{C}) = \frac{1}{\alpha_{\max}} \int_{0}^{\alpha_{\max}} M(C_\alpha) d\alpha,$$  

(10)

where $\alpha_{\max}$ is a maximal value on $\hat{C}$.

4. Estimation the group of students at bilingual education by fuzzy inferences method

As is known, today one of effective methods of managerial technologies is elements of artificial intelligence including fuzzy logic and fuzzy processors, which well proved in decision-making (Zadeh, 2001). In particular, application of methods of fuzzy logic in the cognitive networks management allows to consider easily a set of parameters for decision-making and doesn't demand difficult mathematical calculations (Zadeh, 1974). Moreover, the mathematical apparatus of the fuzzy sets theory allows to operate equally easily both metrizable and nonmetrizable data (Zadeh, 1976).
Rzayev (2013) presented in some detail the problem of point estimation of alternatives under fuzziness of available information. On the basis of the application of this methodology let us obtain the estimation of bilingual lesson from the point of view of evaluation of the sub-groups in whole and students engaged in outside groups on special assignment.

So, suppose that in some academic group of bilingual education during the classes in a particular general discipline a Methodist from the monitoring team conducted their observations of the behavior (activity) of students in subgroups and ordered its estimates on a ten-point scale in the type of Table 4. In this case the subgroups of students are alternatives that are denoted by \( a_1, a_2, a_3 \) and \( a_4 \).

Table 4. Estimation the student activities in subgroups

<table>
<thead>
<tr>
<th>Groups</th>
<th>Speak working (ESSENTIALLY)</th>
<th>Work with the material (INTENSIVELY)</th>
<th>Read, write (PRODUCTIVELY)</th>
<th>Watch, listen (CAREFULLY)</th>
<th>Do nothing (FREQUENTLY)</th>
<th>Wait for assistance (PERMANENTLY)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( a_1 )</td>
<td>2</td>
<td>10</td>
<td>7</td>
<td>8</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>( a_2 )</td>
<td>5</td>
<td>8</td>
<td>6</td>
<td>10</td>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td>( a_3 )</td>
<td>8</td>
<td>5</td>
<td>5</td>
<td>7</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>( a_4 )</td>
<td>10</td>
<td>3</td>
<td>2</td>
<td>5</td>
<td>7</td>
<td>8</td>
</tr>
</tbody>
</table>

Then for numerical (point) estimation the activity of subgroups in bilingual education let’s choose as basis the following consistent reasoning:

\( e_1 \): “If students working in the subgroups talk in essence and at the same time work with the didactic materials, and, if necessary, look at teacher and listen to him, then their activity during classes is satisfactory”;

\( e_2 \): “If in addition to the above observations students all over the classes rarely idle and do not expect any help from the outside, then their activity during the classes is more than satisfactory”;

\( e_3 \): “If in addition to the conditions specified in \( e_2 \) students within the subgroups
alternately read and write, then their academic activity is perfect”;

*e4*: “If students working in the subgroups talk in essence and at the same time work with the didactic materials, alternately read and write and, if necessary, look at teacher and listen to him, during the classes rarely idle, but resort to the help of outside, then their academic activity is very satisfactory”;

*e5*: “If students working in the subgroups talk in essence and at the same time work with the didactic materials, read and write productively and do not always look at teacher and listen to him, and during the classes rarely idle, but often resort to the help of outside, then their activity during classes is satisfactory”;

*e6*: “If students within the subgroups do not work with the didactic materials, do not look at teacher and listen to him and do not idle, then their academic activity is unsatisfactory”.

In formulating these arguments were used six criteria, which used as the values of the corresponding input linguistic variables $x_k$ ($k=1\div6$) for multi-criteria evaluation of the students activities in the academic subgroups. The result of this estimation is one of the values of the output linguistic variable “academic activity” ($Y$).

So, based on the terms of designated linguistic variables let’s reformulate the above reasoning in the form of following implication rules:

*e1*: “If $X_1=\text{ESSENTIALLY}$ and $X_2=\text{INTENSIVELY}$ and $X_4=\text{CAREFULLY}$, then $Y=\text{SATISFACTORY}$”;

*e2*: “If $X_1=\text{ESSENTIALLY}$ and $X_2=\text{INTENSIVELY}$ and $X_4=\text{CAREFULLY}$ and $X_5=\text{RARELY}$ and $X_6=\text{NOT PERMANENTLY}$, then $Y=\text{MORE THAN SATISFACTORY}$”;

*e3*: “If $X_1=\text{ESSENTIALLY}$ and $X_2=\text{INTENSIVELY}$ and $X_3=\text{PRODUCTIVELY}$ and $X_4=\text{CAREFULLY}$ and $X_5=\text{RARELY}$ and $X_6=\text{NOT PERMANENTLY}$, then $Y=\text{PERFECT}$”;

*e4*: “If $X_1=\text{ESSENTIALLY}$ and $X_2=\text{INTENSIVELY}$ and $X_3=\text{PRODUCTIVELY}$ and $X_4=\text{CAREFULLY}$ and $X_5=\text{RARELY}$ and
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As the universe for fuzzy subsets which describe the values of the output linguistic variable Y let’s choose a discrete set J={0, 0.1, 0.2, ..., 1}, and as membership functions which reduce these fuzzy sets let’s choose the following functions (Rzayev, 2013):

- $\tilde{S}$ =Satisfactory as: $\mu_{\tilde{S}}(x) = x, x \in J$;
- $M\tilde{S}$ =More than satisfactory as: $\mu_{M\tilde{S}}(x) = \sqrt{x}, x \in J$;
- $\tilde{P}$ =Perfect as: $\mu_{\tilde{P}}(x) = \begin{cases} 1, & x = 1, x \in J; \\ 0, & x < 1, x \in J; \end{cases}$
- $\tilde{V}\tilde{S}$ =Very Satisfactory as: $\mu_{\tilde{V}\tilde{S}}(x) = x^2, x \in J$;
- $U\tilde{S}$ =Unsatisfactory as: $\mu_{U\tilde{S}}(x) = 1 - x, x \in J$.

Fuzzification of terms from the left parts of the rules adopted by using Gaussian membership functions

$\mu(u) = \exp\left(-\frac{(u - 10)^2}{\sigma_k^2}\right)$ (k=1-6) (see Fig. 1), which reduce the fuzzy sets on the support vector $(a_1, a_2, a_3, a_4)$, and values for the $\sigma_k$ are selected on the basis of the importance degree of the criterion of bilingual education quality.
Thus, the estimation criteria of bilingual education in the academic subgroups let’s define by the following fuzzy sets:

**ESSENTIALLY** (speak working)

\[
\tilde{A} = \frac{0.0183}{a_1} + \frac{0.2096}{a_2} + \frac{0.7788}{a_3} + \frac{1}{a_4};
\]

**INTENSIVELY** (work with the material)

\[
\tilde{B} = \frac{1}{a_1} + \frac{0.6412}{a_2} + \frac{0.0622}{a_3} + \frac{0.0043}{a_4};
\]

**PRODUCTIVELY** (read, write)

\[
\tilde{C} = \frac{0.3679}{a_1} + \frac{0.1690}{a_2} + \frac{0.0622}{a_3} + \frac{0.0008}{a_4};
\]

**CAREFULLY** (watch, listen)

\[
\tilde{D} = \frac{0.8521}{a_1} + \frac{1}{a_2} + \frac{0.6977}{a_3} + \frac{0.3679}{a_4};
\]

**FREQUENTLY** (do nothing)

\[
\tilde{E} = \frac{0.0622}{a_1} + \frac{0.0622}{a_2} + \frac{0.3679}{a_3} + \frac{0.7788}{a_4};
\]

**PERMANENTLY** (wait for assistance from outside)

\[
\tilde{F} = \frac{0.1299}{a_1} + \frac{0.2709}{a_2} + \frac{0.6004}{a_3} + \frac{0.9216}{a_4}.
\]

Then, using these formalisms let’s formulate fuzzy rules as:

e1: «If \( X_1=\tilde{A} \) and \( X_2=\tilde{B} \) and \( X_4=\tilde{D} \), then \( Y=\tilde{S} \); 

\[
e_2: \text{ «If } X_1=\tilde{A} \text{ and } X_2=\tilde{B} \text{ and } X_4=\tilde{D} \text{ and } X_5=\tilde{E} \text{ and } X_6=\tilde{F} \text{, then } Y=M\tilde{S} \); 

\[
e_3: \text{ «If } X_1=\tilde{A} \text{ and } X_2=\tilde{B} \text{ and } X_3=\tilde{C} \text{ and } X_4=\tilde{D} \text{ and } X_5=\tilde{E} \text{ and } X_6=\tilde{F} \text{, then } Y=\tilde{P} \); 

\[
e_4: \text{ «If } X_1=\tilde{A} \text{ and } X_2=\tilde{B} \text{ and } X_3=\tilde{C} \text{ and } X_4=\tilde{D} \text{ and } X_5=\tilde{E} \text{ and } X_6=\not\tilde{F} \text{, then } Y=V\tilde{S} \);
\]
es: «If $X_1=\tilde{A}$ and $X_2=\tilde{B}$ and $X_3=\tilde{C}$ and $X_4=-\tilde{D}$ and $X_5=\tilde{E}$ and $X_6=-\tilde{F}$, then $Y=\tilde{S}$ »;

e_6: «If $X_2=-\tilde{B}$ and $X_4=-\tilde{D}$ and $X_5=-\tilde{E}$, then $Y=US\tilde{S}$ ».

Further, for the left parts of these rules let’s compute the membership function $\mu_{\tilde{A}_i}(u)$ ($i=1+6$). In particular, we have:

e_1: $\mu_{\tilde{A}_1}(a) = \min \{\mu_{\tilde{A}}(a), \mu_{\tilde{B}}(a), \mu_{\tilde{C}}(a)\}$

$$\tilde{M}_1 = \frac{0.0183}{a_1} + \frac{0.2096}{a_2} + \frac{0.0622}{a_3} + \frac{0.0043}{a_4}$$

$$e_2: \mu_{\tilde{M}_2}(a) = \min \{\mu_{\tilde{A}}(a), \mu_{\tilde{B}}(a), \mu_{\tilde{C}}(a), \mu_{\tilde{D}}(a), \mu_{\tilde{E}}(a), \mu_{\tilde{F}}(a)\}$$

$$\tilde{M}_2 = \frac{0.0183}{a_1} + \frac{0.0622}{a_2} + \frac{0.6622}{a_3} + \frac{0.0043}{a_4}$$

e_3: $\mu_{\tilde{M}_3}(a) = \min \{\mu_{\tilde{A}}(a), \mu_{\tilde{B}}(a), \mu_{\tilde{C}}(a), \mu_{\tilde{D}}(a), \mu_{\tilde{E}}(a), \mu_{\tilde{F}}(a)\}$

$$\tilde{M}_3 = \frac{0.018}{a_1} + \frac{0.062}{a_2} + \frac{0.062}{a_3} + \frac{0.0008}{a_4}$$

$$e_4: \mu_{\tilde{M}_4}(a) = \min \{\mu_{\tilde{A}}(a), \mu_{\tilde{B}}(a), \mu_{\tilde{C}}(a), \mu_{\tilde{D}}(a), \mu_{\tilde{E}}(a), \mu_{\tilde{F}}(a)\}$$

$$\tilde{M}_4 = \frac{0.0183}{a_1} + \frac{0.0622}{a_2} + \frac{0.0622}{a_3} + \frac{0.0008}{a_4}$$

As a result, the rules can be written in a more compact form:

e_1: «If $X=M\tilde{A}_1$, then $Y=\tilde{S}$ »;

e_2: «If $X=M\tilde{A}_2$, then $Y=M\tilde{S}$ »;

e_3: «If $X=M\tilde{A}_3$, then $Y=\tilde{P}$ »;

To convert these rules we use the Lukasiewicz’s implication (Rzayev, 2013).

Then, for each pair $(u,j)\in U\times Y$ on $U\times Y$ one can obtain the following fuzzy relations

$$\tilde{M}_5 = \frac{0.0183}{a_1} + \frac{0.0622}{a_2} + \frac{0.0622}{a_3} + \frac{0.0008}{a_4}$$

$$\tilde{M}_6 = \frac{0.0183}{a_1} + \frac{0.0622}{a_2} + \frac{0.3023}{a_3} + \frac{0.2212}{a_4}$$

As a result of the intersection of relations $R_1$, $R_2$, ..., $R_6$ one can obtain overall functional solution:

To find the point estimates of $a_k$ ($k=1-4$) let's apply the rule of composite bilingual education in the academic subgroups reference in the fuzzy environment:
\[ \tilde{E}_k = \tilde{G}_k \circ R, \] where \( \tilde{E}_k \) is the fuzzy interpretation of the assessment, \( \tilde{G}_k \) is the mapping of \( k \)-th estimation in the form of a fuzzy subset on \( U \). Then, according to Rzayev (2013), we have:

\[
\mu_{\tilde{E}_k}(j) = \max_u \left\{ \min(\mu_{\tilde{G}_k}(a), \mu_R(a)) \right\},
\]
where \( \mu_{\tilde{G}_k}(a) = \begin{cases} 0, & a \neq a_k; \\ 1, & a = a_k. \end{cases} \) It follows that

\[
\mu_{\tilde{E}_k}(j) = \mu_R(a_k, j), \text{i.e. } \tilde{E}_k \text{ is a } k \text{-th row of the matrix } R.
\]

Now one can apply the above procedure to obtain the point estimation of bilingual education in subgroups. So, for the first subgroup \( a_1 \) we have estimation in the form of following fuzzy set:

\[
\tilde{E}_1 = \frac{0.9817}{0} + \frac{0.9817}{0.1} + \frac{0.9817}{0.2} + \frac{0.9817}{0.3} + \frac{0.9817}{0.4} + \frac{0.9817}{0.9} + \frac{1}{1.0}.
\]

Calculating its level sets \( E_{ja} \) and corresponding cardinal number \( M(E_{ja}) \) according to the formula:

\[
M(C_a) = \sum_{j=1}^n \frac{i_j}{n}, \quad i \in C_a,
\]
we have:

- for \( 0 < \alpha < 0.9817 \): \( \Delta \alpha = 0.9817 \), \( E_{1a} = \{0; 0.1; 0.2; 0.3; 0.4; 0.5; 0.6; 0.7; 0.8; 0.9; 1\} \), \( M(E_{1a}) = 0.50 \);
- for \( 0.9817 < \alpha < 1 \): \( \Delta \alpha = 0.0183 \), \( E_{1a} = \{1\} \), \( M(E_{1a}) = 1 \).

Further, using the formula

\[
F(\tilde{C}) = \frac{1}{a_{\max}} \int_0^{a_{\max}} M(C_a) \, d\alpha \tag{Rzayev, 2013}
\]

one can find a point estimation of sufficiency of bilingual education in the first subgroup: \( F(\tilde{E}_1) = 0.5092 \).

By similar computations one can find the point estimates of bilingual classes for other subgroups: for \( a_2 \) – \( F(\tilde{E}_2) = 0.5408 \); for \( a_3 \) – \( F(\tilde{E}_3) = 0.4776 \); for \( a_4 \) – \( F(\tilde{E}_4) = 0.4821 \). The best in the bilingual classes is \( a_2 \), which corresponds to the highest point estimate of 0.5408. Next: \( a_1 \rightarrow 0.5092; a_4 \rightarrow 0.4821; a_3 \rightarrow 0.4776 \).

Now let us consider and evaluate observations another Methodist, who during the bilingual classes fixes: how did work students which have a special assignment? His observations are summarized in the following table.
Table 5. Estimation the work of students having a special assignment

<table>
<thead>
<tr>
<th>Criterion</th>
<th>Listen to teacher</th>
<th>Busy</th>
<th>Wait for help of the teacher</th>
<th>Dream, act the goat, idle</th>
</tr>
</thead>
<tbody>
<tr>
<td>The number of student</td>
<td>6 of 10</td>
<td>8 of 10</td>
<td>7 of 10</td>
<td>2 of 10</td>
</tr>
</tbody>
</table>

To assess the designated category of students in bilingual classes let’s use the following trivial, but the consistent and objective statements:

\( e_1 \): “If during the bilingual classes the number of students listens to teacher is a small, the total number of busies is low, many wait and resort to the help of teacher permanently, and the number of inactive is significant, then the quality of bilingual lessons is unsatisfactory”;

\( e_2 \): “If the number of students listens to teacher is a half, the total number of busies is more than half, not many wait and resort to the help of teacher, and the number of inactive is a small, then the quality of bilingual lessons is satisfactory”;

\( e_3 \): “If the number of students listens to teacher is majority, the total number of busies is majority, only some wait and resort to the help of teacher permanently, and the number of inactive is a small, then the quality of bilingual lessons is more than satisfactory”;

\( e_4 \): “If all students listens to teacher, the total number of busies is maximal, the students wait and resort to the help of teacher only in exceptional case, and inactive students is absent, then the quality of bilingual lessons is perfect”;

\( e_5 \): “If the number of students listens to teacher is majority, the total number of busies is more than half, some wait and resort to the help of teacher sufficiently frequently, and inactive students is absent, then the quality of bilingual lessons is very satisfactory”;

\( e_6 \): “If the number of students listens to teacher is majority, the total number of busies is a half, some of students wait and resort to the help of teacher sufficiently frequently, and the number of inactive is a
small, then the quality of bilingual lessons is satisfactory”.

Taking these statements as a verbal estimation model of students learning in the individual program, as input characteristics we assume the terms of appropriate linguistic variables. For example,

- SMALL, HALF, MAJORITY, ALL are terms of linguistic variable “the number of students listens to teacher” ($X_1$);
- LOW, HALF, MAJORITY, MAXIMAL are terms of linguistic variable “the number of busies” ($X_2$);
- MANY, NOT MANY, SOME, IN EXCEPTIONAL CASE are terms of linguistic variable “waiting and resorting to the help of teacher” ($X_3$);
- SIGNIFICANT, SMALL, ABSENT are terms of linguistic variable “the number of inactive” ($X_4$).

Considering the linguistic variable “quality of bilingual lessons” ($Y$) as output characteristics of model, possessing the value (terms):

- SATISFACTORY,
- MORE THAN SATISFACTORY,
- UNSATISFACTORY,
- PERFECT,
- VERY SATISFACTORY,

one can rewrite the above mentioned statements (verbal model) as following implicative rules:

$e_1$: “If $X_1=$ SMALL and $X_2=$ LOW and $X_3=$ MANY and $X_4=$ SIGNIFICANT, then $Y=$ UNSATISFACTORY”;

$e_2$: “If $X_1=$ HALF and $X_2=$ MAJORITY and $X_3=$ NOT MANY and $X_4=$ SMALL, then $Y=$ SATISFACTORY”;

$e_3$: “If $X_1=$ MAJORITY and $X_2=$ MAXIMAL and $X_3=$ SOME and $X_4=$ SMALL, then $Y=$ MORE THAN SATISFACTORY”;

$e_4$: “If $X_1=$ ALL and $X_2=$ MAXIMAL and $X_3=$ IN EXCEPTIONAL CASE and $X_4=$ ABSENT, then $Y=$ PERFECT”;

$e_5$: “If $X_1=$ MAJORITY and $X_2=$ MAJORITY and $X_3=$ SOME and $X_4=$ ABSENT, then $Y=$ VERY SATISFACTORY”;

$e_6$: “If $X_1=$ MAJORITY and $X_2=$ HALF and $X_3=$ SOME and $X_4=$ SMALL, then $Y=$ SATISFACTORY”.
These rules have been realized in the notation of MATLAB\Fuzzy Inferences Systems (Fig. 2). For the fixed observation (Table 5) on the scale of the interval [0,1] it was obtained the numerical estimation (0.483) of bilingual lesson from the point of view of busies outside the subgroups in accordance with a special program.

![Figure 2. Estimation of bilingual lesson in the notation of MATLAB\Fuzzy Inferences Systems](image)

**Result**

Currently bilingual education is very actual problem in the countries, where a large number of migrants, for example, in USA and EU. Proposed paper formulates and tests a new approach to the assessment of bilingual education, which is based on the fuzzy inferences method. For an alternative assessment of bilingual classes it is selected block of evaluation criteria by which on the basis of correspondent marks of observations made assessment of audience as a whole. According to the results of bilingual classes it is possible to obtain estimates of students (individually) and teacher by alternative computations.

It is quite obvious that the system of bilingual education should be flexible, i.e. it must develop continuously by introduction the control system of the quality of student
learning. This paper proposes a new approach for estimating the bilingual classes based on the application of fuzzy inference mechanism to estimate the audience as a whole, concrete student during the integral classes and a teacher. This approach allows to adapt the verbal model to different conditions and, most importantly, to use of the existing instructional lines in the field of bilingual education.
References


